Inventory allocation when demand is private knowledge: the impact of allocation mechanisms on retailers' strategic ordering

Eirini Spiliotopoulou

1 Motivation

When a number of retailers that trade a common product are serviced from a single warehouse (e.g., a central distribution center), the issue of how inventory is allocated to them becomes very important. In the absence of a pricing mechanism that would balance supply and demand, inventory rationing through quantity limits (upper or lower) in case of shortage or surplus are necessary. Allocation, or rationing of limited common inventory, is an ongoing issue in many industries where there is only one opportunity for production or procurement, before the start of the selling season. For example, allocation mechanisms have been employed in the fashion apparel, consumer electronics and, automotive industries (Cachon and Lariviere, 1999a,b; Lu and Lariviere, 2012). In such cases, retailers may have an incentive to misreport their needs in order to gain a more favorable allocation. Many supply chain failures have been caused by the ordering behavior of downstream parties in particular when order inflation occurs due to capacity shortage) (Cachon and Lariviere, 1999c; Lu and Lariviere, 2012).

The issue of capacity rationing in a supply chain with one supplier selling to multiple retailers is a phenomenon well-studied in the literature (Cachon and Lariviere, 1999a,b,c). When several retailers compete for limited capacity, a broad class of allocation mechanisms are prone to manipulation: retailers may order more than needed to gain a higher allocation (Cachon and Lariviere, 1999c). We consider instead the case of multiple retailers who pool their inventory to more efficiently satisfy local demands, or equivalently, the case of a single company that services a common product across multiple stores from a central distribution center. In the case of a pooling coalition, this is managed centrally by a single entity, the central planner (CP).

Our setting differs from prior capacity rationing research in capacity rationing three ways: (a) the manager of the pooling coalition (CP) is interested in maximizing total supply chain profits (she is not a separate business entity trying to maximize her own profits), (b) when allocation takes place, retailers already know their realized demands (not their optimal stocking levels under demand uncertainty), and (c) retailers are responsible for both the under-stocking and over-stocking costs of the coalition. In contrast to the case of capacity rationing, total inventory of the coalition is held in the central warehouse until local demands are realized. After demands are realized and final orders are placed to the central warehouse, retailers may receive less or more than their final order. Retailers collectively assume all demand uncertainty risk, as the central planner is not a separate unit with its own financial objectives.

Previous behavioral research on retailers' ordering behavior when capacity is binding shows that Nash equilibrium predictions substantially exaggerate retailers tendency to strategically order more than they need (Chen et al., 2012). Chen et al. (2012) consider a supply chain with two retailers and a single supplier, where, if orders exceed the suppliers capacity, quantities are allocated proportionally to the orders. We consider various allocation mechanisms and we want to study their impact on retailers' ordering, both analytically and experimentally, in the context of an integrated supply chain. We study three allocation mechanisms that are commonly used in practice and analyzed in the literature: proportional, linear and uniform.

2 Research questions

We study the ordering behavior of retailers under various allocation mechanisms and focus on the quality of demand information passed to the central system through final orders. More specifically, we wish to answer the following questions:

- Which allocation mechanisms (i.e., proportional, linear and uniform) are Pareto optimal when local demands are common knowledge?
- When realized demands remain retailers' private knowledge, under which allocation schemes will retailers truthfully report their private demand information to the central system who allocates inventory? Do Pareto optimal allocation mechanisms exist that are also truth-inducing?
- Does strategic ordering (e.g., frequency, magnitude), in practice, depend on the allocation mechanism? Under which allocation mechanism is order inflation (deflation) more pronounced?

- Does the inventory level impact retailers' level of distortion?

3 Setting

A number of retailers n have decided to form a pooling coalition to better satisfy their uncertain demand. Retailers are symmetric regarding cost and revenue parameters as well as the demand distribution they face. The also enjoy local monopolies (e.g., they each serve a different geographic region with separate customer bases). The level of inventory held for the coalition is assumed to be a given quantity (i.e., it is a parameter) and common knowledge to all players.

After each retailer observes his realized demand, he places an order ($q_i \geq 0$) to the central warehouse. Total inventory is distributed to retailers based on the publicly announced mechanism. We denote by $\boldsymbol{\alpha}$ the vector of quantities sent to retailers. We note that when a retailer places an order, he does not know the realized demands of the other players and thus does not know if there is inventory shortage or surplus. The timing of the events, is as follows:

1. Total Inventory (Q) and the allocation mechanism are announced.
2. Each retailer i learn his (and only his) final demand (d_i).
3. Retailers simultaneously submit their orders (q_i's).
4. Orders are filled according to the posted allocation mechanism and profits are calculated.

We study three allocation rules; *proportional*, *linear* and *uniform*. We build on the allocation rules for capacity rationing proposed by Cachon and Lariviere (1999) and we modify them to fit our setting. Total inventory is assigned to retailers even when inventory exceeds total demand. Perhaps the most intuitive mechanism is the proportional allocation where each retailer receives a proportional amount of his order. Linear allocation gives each retailer his order plus (minus) a common quantity when there is inventory surplus (shortage). Under uniform allocation, each retailer gets the same quantity, under some conditions. No retailer gets more than what he asks for when total orders exceed total inventory and no retailer gets less than what he asks when the reverse is true. Under common knowledge, these conditions guarantee that there are no unsold units when there is inventory shortage and that there is no unmet demand when there is inventory surplus.

4 Analytical results and hypotheses

We first show that under common knowledge, all three allocation rules are Pareto optimal. When based on realized demands at each retail location, these allocation mechanisms guarantee that we cannot have at the same time inventory shortage at one location and surplus at another. The allocation of Q across regions is efficient, maximizing total profit. This implies that if retailers place an order equal to their demand these allocation functions are Pareto optimal. But it is unclear whether retailers will truthfully report their realized demands to the central system (i.e. whether $q_i = d_i \; \forall i$).

To answer this question, we use the concept of dominant strategy equilibrium. We define $q_i(d_i)$ to be a function mapping from $[\underline{d_i}, \overline{d_i}]$ to $[0, Q]$. This function defines a strategy for player i, dictating an order for each possible demand realization. Similarly, q_{-i} denotes the vector of orders submitted by all retailers but retailer i. The function $q^*(d) = \{q_1^*(d_1), q_2^*(d_2), ..., q_n^*(d_n)\}$ forms a dominant equilibrium for all i and $\mathbf{d}$, *if and only if*:

$$\pi_i(\alpha_i(q_i^*(d_i), q_{-i}(d_{-i}))) \geq \pi_i(\alpha_i(q_i(d_i), q_{-i}(d_{-i}))) \; \forall \; q_i, q_{-i} \in [0, Q] \tag{1}$$

In a dominant strategy equilibrium, each retailer has a strategy that maximizes his profit regardless of the strategies of the other retailers. We are interested in strategies where retailers order their optimal quantities, their true needs, in a dominant equilibrium ($q_i^*(d_i) = d_i$). We have the following results.

Proposition 1: Truthfully reporting their realized demands is not a dominant strategy equilibrium for retailers under the proportional or the linear allocation mechanism.

Proposition 2: Truthfully reporting their realized demands is a dominant strategy equilibrium for retailers under the uniform allocation mechanism.

The intuition behind this result is that when the uniform allocation rule is employed and there is inventory shortage, a retailer can increase his allocation by untruthfully placing an order above his demand only when he belongs to those that receive an allocation equal to their final order. Motivated by these results we formulate the following hypotheses:

Hypothesis 1: The uniform allocation mechanism will result in less retailer misreporting compared with the proportional allocation mechanism (i.e., retailers will less often report $q_i \neq d_i$).

Hypothesis 2: The uniform allocation mechanism will result in less retailer misreporting compared with the linear allocation mechanism (i.e., retailers will less often report $q_i \neq d_i$).

Hypothesis 3: Total profits are lower under the proportional and the linear allocation rules compared to the case of uniform allocation.

We would also like to explore the impact of inventory level on retailers' ordering behavior and incentives to misreport their demand. When inventory is lower, other things being equal, the probability of inventory shortage is higher. Also, the average magnitude of shortage will be higher. Under a proportional or linear allocation rule, retailers may increase their orders to gain a higher allocation.

Hypothesis 4: Lower inventory incentivizes retailers to inflate their orders above real demand more and more often when the proportional or the linear allocation mechanism is employed.

5 Experimental Design

We propose a controlled laboratory experiment to test these predictions. We use a between subjects, 2 x 3 experimental design, as described schematically in Table 1.

	Allocation		
Inventory	Proportional	Linear	Uniform
High	Case 1	Case 2	Case 3
Low	Case 4	Case 5	Case 6

Table 1: Experimental Design

Each session (corresponding to one case) consists of multiple rounds and subjects are randomly matched in each round. Comparing cases 1 and 2 with 3 will allow us to test hypotheses 1 and 2, respectively. Also, comparing the total profits generated under the three allocation rules (in each round) we can test hypothesis 3. To keep profit comparisons fair, we will use the same demand streams across treatments (generated before the experiment according to the commonly known demand distribution at each retail location). Under cases 4, 5 and 6, inventory level is low enough so that shortage is certain. By comparing cases 4, 5, and 6 against 1, 2, and 3 we can test the effect of higher probability of inventory shortage on retailers' ordering and how this effect is moderated by the allocation mechanism employed (to test hypothesis 4).

6 Summary

Acknowledging that the issue of allocation is present and may play an important role in the dynamics of a pooling coalition formed by multiple retailers, we study the impact of three well-known allocation mechanisms on retailers' strategic ordering behavior and resulting profits. We first show analytically that under common knowledge the proportional, linear and uniform allocation schemes we are considering are all Pareto efficient. But, under asymmetric information (allocation is based on retailers' orders), only the uniform rule induces truth-telling and results in Pareto optimal outcomes. Under this allocation scheme, retailers have an incentive to truthfully report their realized demands by placing a final order equal to local demand (dominant strategy equilibrium). Previous research has shown, in another context, that standard game theory predictions substantially exaggerate retailers' strategic ordering. We therefore want to further explore how the various allocation mechanisms may affect order placement in practice. To do so, we propose a controlled laboratory experiment to identify additional behavioral factors that potentially affect decision making in this context (e.g., bounded rationality, trustworthiness, fairness concerns).

References

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