Reference price as a new dimension of demand estimation in revenue management

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Abstract

Nowadays customers are exposed to a lot of information regarding prices such as promotions, discounts and pricing policies. With few clicks in the internet, everyone is able to scan easily the price of different products through different selling channels. The ability of customers to extract detailed information about previous sales forms an unwritten expectation that impacts their willingness to pay. In this paper, we consider reference price as a new dimension of demand estimation in revenue management. When this new dimension is added to customer decision process, the firm not only needs to control the remaining capacity but also the reference price in order to maximize the expected profit. We show that due to some changes in the structural properties of value function such as monotonicity and modularity, the effectiveness of booking limit control policy declines dramatically. However, under certain assumptions including the Poisson arrival process of customers, bid price policy still leads to the optimal solution, although with a different price threshold. Toward the completion of this paper, we develop a comprehensive simulator to study and illustrate our results and to compare its performance with previous methods. We show that the expected revenue may increase up to more than 7 percent using the model developed in this paper.

1. Introduction

The revenue management problem wherein a firm controls the inventory of a given product through fare classes is well studied in literature. Different aspects such as demand management and competitive models are studied and readers are referred to Talluri and van Ryzin (2004) for an extensive review. Although the revenue management literature is vast and comprehensive, its interaction with other fields such as marketing, finance, and strategic management is missing. Customer psychology and its impact on operation management is less studied and dates back to Aviv and Pazgal (2008) that studies optimal dynamic pricing in the presence of strategic customers. They use a single price reduction in a fixed point in finite period and show that losses due to ignoring strategic customers are significant. Elmaghraby et al. (2008) expand this research and consider multiple price changes. Su (2007) studies the impact of strategic customers in the presence of price variation over time. This research extends previous papers and let the price changes to happen in any direction. Su (2009), Levin et al. (2010), Zhao et al. (2012) and Nasiry and Popescu (2011, 2012) also study different aspects of customer behavior in dynamic pricing. In revenue management the process of customer choice are studied in Talluri and van Ryzin (2005), Anderson and Wilson (2003), Shen and Su (2007) and Mei et al. (2012).

One of the aspects of customer psychology is reference price. With the tremendous growth of internet application, advertisement technology, and the rise of advanced online selling, customers are able to find past and current prices for the same or similar products that they plan to purchase especially in travel industry. Customers build expectation about price levels herein reference price that impacts their willingness to pay. However, customers are different on how they react to the difference between the posted price and their expectation. This heterogeneity among customers is measured by customers' "loss aversion." A loss neutral customer assigns the same weight to losses and gains when his expectation differs the posted price. For a loss averse customer the losses have higher impact compared with the same amount of gain. A loss seeker feels otherwise. The literature of loss aversion is comprehensive (see Kahneman and Tverski (1979), Tverski and Kahneman (1992), Nasiri and Popescu (2011) and literature there). Kahneman and Tversky (1979) show that by neglecting reference price firms might set prices too low resulting in profit losses. Based on this analysis they introduce the prospect theory that is a seminal result in reference-dependent preference literature. Popescu and Wu (2007), Winer (1985) and Greenleaf (1995) investigate the impact of reference price on promotions and pricing policies. Fibich Gadi et al. (2007) analyze the effect of price promotion in the presence of asymmetric reference price. They show that promotions would lead to profit decline when the loss has stronger impact on demand. Nasiry and Popescu (2011) study dynamic pricing in the presence of reference price based on the peak-end rule. More details about reference price formation can be found in Mazumdar et al. (2005).

Although customer behavior is being considered in revenue management literature, to the best of our knowledge, no publicly-available research considers reference prices in specific. In revenue management with reference price the firm not only needs to control its inventory but also needs to control the reference price as it directly impacts the demand. In this paper we show that when reference price exists some of traditional properties do not hold anymore. Without those monotonicity properties the existing heuristics such as the booking limit control do not necessarily represent a good estimation of optimal policy with acceptable performance. We illustrate this claim using a numerical analysis and show how ineffective the heuristics can be in different circumstances. For the new model, we provide an optimal solution and update the existing heuristics considering reference price.

2. Model

Consider a firm selling *N* identical products such as airline tickets or hotel rooms over a finite time horizon. The horizon is discretized such that no more than one customer arrives in one period. The time runs forward that is $t = 0$ refers to the beginning of the horizon and $t = T - 1$ refers to the last period when a sale may occur, and in $t = T$ no decisions are made but the boundary conditions apply. The pricing of the products is fare-based and there are m fare classes where the fare prices are pre-determined. The fare classes are ordered based on the price levels and that $f_1 \geq f_2 \geq \cdots \geq f_m$, where f_i is the fare price of class *i*.

Reference price

Customers form a reference price for each product based on the price history. Let's define r_t as the reference price of the customers in period *t* that is identical among all customers. We model the reference price using the weighted average of past prices, with more weight to be given to recent prices, which is a commonly used in literature (See Mazumdar et al. (2005), Popescu and Wu (2007) and Nerlove (1958)).

$$r_t = \alpha r_{t-1} + (1-\alpha) f_{t-1}, for\ \alpha \in [0.1) \qquad (1)$$

where the reference price vary over the price range and that $f_m \leq r_t \leq f_1$, for any $t \leq T$. Customers' arrival rate for each class not only depends on the product price but also on the reference price. The probability of a class *i* arrival in period *t* when the reference price is r_t (or *r* for short) is defined by $P_i(t,r)$ where $P_0(t,r)$ denotes the probability of no arrival. We therefore have $P_0(t,r) + \sum_{j=1}^{m} P_j(t,r) = 1$. Let's assume that the arrival rate of class *j* customers follows Poisson with rate $\lambda_j(t,r)$ that is a time dependent process as a function of the reference price *r*. Now, based on Nasiry and Popescu (2011) we assume that the rate of arrival is calculated using the following equation.

$$\lambda_j(t,r) = \lambda_j^0(t) - \beta\left(f_j - r\right)^+ + \gamma\left(r - f_j\right)^+ \qquad (2)$$

where $\frac{\lambda_j^0(t)}{f_j - f_m} \geq \beta \geq \gamma \geq 0 \ (1 \leq j \leq m)$ accoding to the loss aversion of customers and $\lambda_j^0(t)$ is arrival rate with no reference price that is unbounded and nonincreasing with respect to the fare level. For the Poisson arrival, we discretize the time periods such that at most one customer per period can arrive in which case the probability of class *j* customer is:

$$P_j(t,r) = \frac{\lambda_j(t,r)}{1 + \sum_{j=1}^{m} \lambda_j(t,r)} \qquad (3)$$

At the beginning of each period, if an arrival occurs the seller needs to decide whether to accept the request or not. This decision is made based on the customer fare class, the number of rooms available *x*, remained time periods *t*, and the impact of the sale on the future reference price, r_{t+1}. Accepting a high fare customer increases the future reference price and the firm decides based on the remaining time and capacity. By accepting a low fare class customer, the risk of having unsold product at the end of sales horizon will be reduced, with the cost of lowering the products' reference price which in turns affect the arrival rate of future customers. There is no decision to be made in the case of no arrival. We formulate this problem using a finite horizon discrete time Markov decision process where the state variables are the total number of remaining products, *x* and the reference price, *r*, and the stage variable is the current time, *t*. We do not consider no-shows, cancellations, and overbookings. Let $u(t) = 1$ if the seller accepts a request (if there exist any) and $u(t) = 0$ otherwise. If the seller accepts the request it earns $R(t)$, a random variable with

$R(t) = fj$ if the arrival is a j class $(j > 0)$ and $R(t) = 0$, otherwise. Upon an acceptance, the reference price gets updated based on Equation (1) and the remaining capacity to $x - 1$. As no cancellation is allowed, the remaining capacity is non-increasing but the reference price can fluctuate within the price range $[f_1, f_m]$ depending on the type of accepted customers. In case of no arrival or a customer rejection the reference price remains unchanged. The maximal expected revenue of selling x products during period t to T is denoted by $V_t(x,r)$ that can be recursively calculated as follows.

$$V_t(x,r) = E\ max_{u\in\{0,1\}}[uR(t) + uV_{t+1}\big(x-1, \alpha r + (1-\alpha)R(t)\big) + (1-u)V_{t+1}(x,r)]$$

with boundary conditions

$$V_T(x,r) = 0 \qquad for\ all\ 0 \le x \le N, f_1 \le r \le f_m$$

$$V_t(0,r) = 0 \qquad for\ all\ 0 \le t \le T, f_1 \le r \le f_m \tag{4}$$

With some rearrangement we have:

$$V_t(x,r) = V_{t+1}(x,r) + \sum_{j=1}^{m} p_j(t,r)\left[f_j + V_{t+1}\big(x-1, \alpha r + (1-\alpha)f_j\big) - V_{t+1}(x,r)\right]^+$$

$with$ the $same$ boundary conditions (5)

Therefore the optimality condition upon receiving a type j request is:

$$f_j \ge V_{t+1}(x,r) - V_{t+1}\big(x-1, \alpha r + (1-\alpha)f_j\big) \tag{6}$$

The right hand side of Equation (6) can be considered as the opportunity cost of accepting type j Customers, $\pi_t^j(x,r)$.

3. Results

Two frequently used mechanisms in revenue management are bid price and booking limit. These mechanisms are justified through monotonicity conditions. Studying the structural properties of value function presented in Equation (5) we analyze the effectiveness of these mechanisms considering reference price effect on demand. Consequently we present the optimal solution. Here is the list of results:

(i) Due to the nonmonotonicity of the new opportunity cost of Equation (6), booking limit policy lose its effectiveness in the presence of reference price.

(ii) Under certain conditions (i.e. monotonicity of $V_t(x,r)$ with respect to *r*), bid price policy leads to the optimal solution, however with a new price threshold which is presented in Equation (7).

$$\pi_t(x,r) = V_{t+1}(x,r) - V_{t+1}(x-1, \alpha r + (1-\alpha) f_k) \qquad where$$

$$k = argmin_j \{f_j \geq V_{t+1}(x,r) - V_{t+1}(x-1, \alpha r + (1-\alpha) f_j)\} \tag{7}$$

(iii) According to the literature, we assume Poisson distribution for customers' arrival. We also assume the time periods to be small enough so that we can ignore the probability of multiple arrival in each period. Using some mathematical modeling we can show "for Poisson arrival of customers and m fare levels, there exists a positive value, *z*, for which the arrival probability of the low-fare customers *j* with $f_j < z$ decreases with respect to *r* and the arrival probability of the high-fare customers *j'* with $f_{j'} > z$ increases in *r*." which means for Poisson arrival of customers and m fare levels, $V_t(x,r)$ is nondecreasing with respect to *r* and therefore bid price policy in Equation (7) leads to the optimal solution.

(iv) We also consider the heterogeneity of customers' fare levels in Equation (2) and show that with a mild assumption, bid price policy still leads to the optimal solution.

We finally use a numerical study based upon a simulation to illustrate our results and in order to highlight the importance of using the reference prices. We consider the classic revenue management model ("Classic" in Table 1) as if the formation of the reference price within customers is ignored and compare it with our model (called "Model" in the table). We refer to Mazumdar et. al. (2005) for parameter selection. We choose α = 0.15, β = 0.0001, and γ = 0.00009. We perform this numerical study when the firm sells x = 75 product over T = 300 periods. There are three classes of customers that arrive according to Poisson process with arrival rates 0.1, 0.2 and 0.4 with fare levels 100, 500 and 1000 respectively. Table 1 shows average reference price, minimum price accepted, remained capacity and revenue at the beginning of different time periods for both the Model and Classic approach. As it is shown the total revenue improvement is 7.5 percent when the firm consider the reference price model compared with the classical revenue management models (comparing 26952 to 28997 in the last column).

Table 1. Average amount of different parameters and variables at the beginning of each time period

	Reference Price		Price Level		Remaining Capacity		Expected Revenue	
T	Model	Classic	Model	Classic	Model	Classic	Model	Classic
300	1000.00	1000.00	500.00	500.00	75.00	75.00	0	0
250	594.61	392.09	500.00	500.00	67.03	59.43	4901	4743
200	598.51	440.24	500.00	500.00	59.18	48.38	9709	9231
150	589.29	427.44	497.61	500.00	51.36	37.36	14467	13793
100	526.51	393.23	410.42	479.61	42.87	25.78	19102	18244
50	418.08	366.78	263.60	375.62	30.11	13.13	23949	22838
20	354.03	330.25	198.80	299.90	20.99	4.69	26887	25629
10	345.38	311.59	185.20	150.40	17.69	1.65	27861	26550
5	316.32	282.11	148.40	150.40	15.92	0.54	28365	26824

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